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Comment on "New pseudoclassical model for Weyl particles"

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Abstract

It is demonstrated that the recently proposed pseudoclassical model for Weyl particles [1] (D.M. Gitman, A.E. Gonçalves and I.V. Tyutin, Phys. Rev. D 50 (1994) 5439) is classically inconsistent. A possible way of removing the classical inconsistency of the model is proposed.

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In a recent paper [1], Gitman, Gonçalves and Tyutin proposed a new pseudoclassical model to describe a Weyl particle. The purpose of the present comment is to demonstrate that classically the model [1] is inconsistent. Namely, it will be shown that the model contains a classical relation of the form $0 = 1$ being a consequence of the Lagrangian constraint, or, equivalently, of the corresponding secondary Hamiltonian constraint. A possible way of removing the classical inconsistency of the model is proposed.

Let us consider the following relation in a superspace with real odd (Grassmann) variables $\psi^a = \psi^{a*}$, $a = 1, 2, \ldots, N$,

$$i \sum_{a=1}^{N} \psi^a \psi^b \omega_{ab} - C \doteq 0. \tag{1}$$

Here ω_{ab} is some constant real antisymmetric c-number matrix and

$$C \neq 0 \tag{2}$$

is a real constant, and the symbol $\doteq$ means ordinary equality when relation (1) is treated as a Lagrangian equation of motion (Lagrangian constraint) and is a weak equality when eq. (1) is considered as a Hamiltonian constraint. One rewrites relation (1) in the equivalent form $iC^{-1} \sum_{a=1}^{N} \psi^a \psi^b \omega_{ab} \doteq 1$. Raising the latter condition to the $[\frac{N}{2} + 1]$th power, $[.]$ being an integer part, one gets on l.h.s. a sum of terms all of them containing factors $(\psi^a)^2 = 0$. Therefore, relation (1) together with condition (2) leads to the relation $0 = 1$, and, therefore, it is self-contradicting.

The model of ref. [1] contains the relations of the form

$$T_\mu = \epsilon_{\mu\nu\lambda\rho} \pi^\nu \psi^\lambda \psi^\rho + i\frac{\alpha}{2} \pi_\mu \doteq 0, \tag{3}$$

$$\pi^2 = \pi_\mu \pi^\mu \doteq 0, \tag{4}$$

$$\pi_\mu \psi^\mu \doteq 0, \tag{5}$$

where π_μ is an even canonical momentum being, simultaneously, the energy-momentum vector of the particle, and $\alpha = +1$ or -1. Taking into account the condition (4), one supplements the light-like vector π_μ with the tetrad components $n^-_\mu(\pi)$, $n^i_\mu(\pi)$, $i = 1, 2$, $n^- \pi = 1$, $\pi n^i = 0$, $n^- n^i = 0$, $n^i n^j = -\delta^{ij}$, forming the complete oriented set: $\pi_\mu n^-_\nu + n^-_\mu \pi_\nu - n^i_\mu n^i_\nu = \eta_{\mu\nu}$, $\epsilon^{\mu\nu\lambda\rho} \pi_\mu n^-_\nu n^1_\lambda n^2_\rho = 1$. If, in addition, one takes into account the constraint (5), the vector set of constraints (3) can be presented equivalently as

$$\pi_\mu \left(-2(\psi n^1)(\psi n^2) + i\frac{\alpha}{2} \right) \doteq 0. \tag{6}$$

From here one concludes that the vector set of constraints (3) is equivalent only to one constraint

$$-2(\psi n^1)(\psi n^2) + i\frac{\alpha}{2} \doteq 0. \tag{7}$$

Due to this fact, there is only one local symmetry in the model [1] in addition to the reparametrization symmetry and the local supersymmetry. Such a symmetry is generated by the constraint (7). But the relation (7) is exactly of the form (1), and one arrives at the conclusion that the model [1] is classically inconsistent.

The problem connected with the constraint (3) can be removed in the following way, which was used earlier in refs. [2] for constructing the pseudoclassical model for planar fermions. Let us extend the model by introducing a pair of scalar mutually conjugate odd variables θ^+ and $\theta^- = (\theta^+)^*$, having the Dirac brackets

$$\{\theta^+, \theta^-\}_D = -i. \tag{8}$$

The corresponding kinetic term in the Lagrangian should be $L^\theta_{kin} = \frac{i}{2}(\theta^+\dot{\theta}^- + \theta^-\dot{\theta}^+)$. Then, if one replaces the c-number parameter α by $\tilde{\alpha} = \alpha\theta^+\theta^-$, the action of model [1] will take the form

$$S = \int_0^1 \left(-\frac{1}{2e}(\dot{x}_\mu - i\psi_\mu\chi - \epsilon_{\mu\nu\lambda\rho}b^\nu\psi^\lambda\psi^\rho + i\frac{\alpha}{2}\theta^+\theta^- b_\mu)^2 - i\psi_\mu\dot{\psi}^\mu + L^\theta_{kin} \right) d\tau, \tag{9}$$

with Lagrange multipliers e and $b_\mu = -b^*_\mu$ (even), and χ (odd). The local symmetries of the system are given by the same transformation laws of the configuration space variables given in ref. [1] and supplemented with the following transformation properties for the variables $\theta^\pm$ with respect to the reparametrization, $\delta\theta^\pm = \dot{\theta}^\pm\xi$, supersymmetry, $\delta\theta^\pm = 0$, and additional symmetry transformations, $\delta\theta^\pm = \mp\frac{1}{2}\alpha e^{-1}zb\theta^\pm\kappa$, where $\xi(\tau)$ and $\kappa(\tau)$ are infinitesimal even parameters, and $z_\mu = \dot{x}_\mu + \ldots$ is the 'elongated' velocity appearing in action (9) as z^2. Action (9) leads to the same set of constraints (3)–(5), but with the only substitution $\alpha \to \tilde{\alpha}$. Such a difference is crucial, since one has the relation $-2(\psi n^1)(\psi n^2) + i\frac{\alpha}{2}\theta^+\theta^- \doteq 0$ instead of the relation (7). This new relation is classically consistent and singles out a nontrivial subspace in the configuration or phase space of the system. Therefore, the described problems with constraint (3) are removed now.

In correspondence with classical relation (8) and $\{\psi_\mu, \theta^\pm\}_D = 0$, the quantum analogs of the odd variables can be realized as $\hat{\psi}_\mu = \frac{i}{2}\gamma_\mu \otimes \sigma_3$, $\hat{\theta}^\pm = \frac{1}{2} \cdot 1 \otimes (\sigma_1 \pm i\sigma_2)$, where one assumes that the operators $\hat{\psi}_\mu$ and $\hat{\theta}^\pm$ act on the wave functions $\Psi(x)$, $\Psi^t = (\psi_u, \psi_d)$, whose components ψ_u and ψ_d are distinguished by the σ-matrix factors in the direct products. Taking for the quantum analog of the classical term $\theta^+\theta^-$ the same ("normal") ordering for noncommuting operators $\hat{\theta}^+$ and $\hat{\theta}^-$, we get $\hat{\tilde{\alpha}} = \alpha\frac{1}{2}1 \otimes (1 + \sigma_3)$. As a result, taking into account the quantum analog of the constraint (5), the constraint (3) (with the described substitution for the parameter α), will turn into the equation $\hat{\pi}_\mu(\gamma^5 \otimes 1 - \alpha\frac{1}{2}1 \otimes (1 + \sigma_3))\Psi = 0$ in correspondence with the classical picture analyzed above. From here one finds that $\psi_d = 0$, and for $\alpha = +1$ and $\alpha = -1$ the component ψ_u will describe the Weyl particle of the corresponding helicity (see ref. [1]). If, instead, one takes $\alpha = \pm 2$ and chooses the antisymmetrized ordering for the operators, $\theta^+\theta^- \to \frac{1}{2}(\hat{\theta}^+\hat{\theta}^- - \hat{\theta}^-\hat{\theta}^+)$, both components ψ_u and ψ_d will survive, and one will have a $P, T-$invariant system of Weyl particles with opposite helicity values.

To conclude, it is worth to note that the same problems with relations of the form (1) are peculiar to different related pseudoclassical models considered in literature [3]. The corresponding models can also be improved with the help of the extension procedure described here.

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